Exercise 4.1 (Revised) - Chapter 4 - Quadratic Equations - Ncert Solutions class 10 - Maths

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NCERT Solutions for Class 10 Maths Chapter 4: Quadratic Equations

Ex 4.1 Question 1.

Check whether the following are Quadratic Equations.
(i) $(x+1)^2=2(x-3)$
(ii) $x^2-2x=(-2)(3-x)$
(iii) $(x-2)(x+1)=(x-1)(x+3)$
(iv) $(x-3)(2x+1)=x(x+5)$
(v) $(2x-1)(x-3)=(x+5)(x-1)$
(vi) $x^2+3x+1=(x-2)^2$
(vii) $(x+2)^3=2x\left(x^2-1\right)$
(viii) $x^3-4x^2-x+1=(x-2)^3$

Answer.

(i) $(x+1)^2=2(x-3)$
$\left\{(a+b)^2=a^2+2ab+b^2\right\}$
$\Rightarrow x^2+1+2x=2x-6$
$\Rightarrow x^2+7=0$

Here, degree of equation is 2 .

Therefore, it is a Quadratic Equation.
(ii) $x^2-2x=(-2)(3-x)$

$\Rightarrow x^2-2x=-6+2x$
$\Rightarrow x^2-2x^{-2x+6}=0$
$\Rightarrow x^2-4x+6=0$

Here, degree of equation is 2 .
Therefore, it is a Quadratic Equation.
(iii) $(x-2)(x+1)=(x-1)(x+3)$
$\Rightarrow x^2+x-2x-2=x^2+3x-x-3=0$
$\Rightarrow x^2+x-2x-2-x^2-3x+x+3=0$
$\Rightarrow x-2x-2-3x+x+3=0$
$\Rightarrow -3x+1=0$

Here, degree of equation is 1 .
Therefore, it is not a Quadratic Equation.
(iv) $(x-3)(2x+1)=x(x+5)$
$\Rightarrow 2x^2+x-6x-3=x^2+5x$
$\Rightarrow 2x^2+x-6x-3-x^2-5x=0$
$\Rightarrow x^2-10x-3=0$

Here, degree of equation is 2 .
Therefore, it is a quadratic equation.
(v) $(2x-1)(x-3)=(x+5)(x-1)$

$\Rightarrow 2x^2-6x-x+3=x^2-x+5x-5$
$\Rightarrow 2x^2-7x+3-x^2+x-5x+5=0$

$\Rightarrow x^2-11x+8=0$

Here, degree of Equation is 2.
Therefore, it is a Quadratic Equation.
(vi) $x^2+3x+1=(x-2)^2$
$\{(a-b)^2=a^2-2ab+b^2\}$
$\Rightarrow x^2+3x+1=x^2+4-4x$
$\Rightarrow x^2+3x+1-x^2+4x-4=0$
$\Rightarrow 7x-3=0$

Here, degree of equation is 1.
Therefore, it is not a Quadratic Equation.
(vii) $(x+2)^3=2x\left(x^2-1\right)$
$\{(a+b)^3=a^3+b^3+3ab(a+b)\}$
$\Rightarrow x^3+2^3+3(x)(2)(x+2)=2x\left(x^2-1\right)$
$\Rightarrow x^3+8+6x(x+2)=2x^3-2x$
$\Rightarrow 2x^3-2x-x^3-8-6x^2-12x=0$
$\Rightarrow x^3-6x^2-14x-8=0$
Here, degree of Equation is 3.

Therefore, it is not a quadratic Equation.
(viii) $x^3-4x^2-x+1=(x-2)^3$

$\{(a-b)^3=a^3-b^3-3ab(a-b)\}$
$\Rightarrow x^3-4x^2-x+1=x^3-2^3-3(x)(2)(x-2)$
$\Rightarrow -4x^2-x+1=-8-6x^2+12x$
$\Rightarrow 2x^2-13x+9=0$

Here, degree of Equation is 2.
Therefore, it is a Quadratic Equation.

Ex 4.1 Question 2.

Represent the following situations in the form of Quadratic Equations:
(i) The area of rectangular plot is $528\ \mathrm{m}^2$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.
(ii) The product of two consecutive numbers is 306 . We need to find the integers.
(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) after 3 years will be 360 . We would like to find Rohan's present age.
(iv) A train travels a distance of $480\ \mathrm{km}$ at uniform speed. If, the speed had been $8\ \mathrm{km/h}$ less, then it would have taken 3 hours more to cover the same distance. We need to find speed of the train.

Answer.

(i) We are given that area of a rectangular plot is $528\ \mathrm{m}^2$.
Let breadth of rectangular plot be x metres
Length is one more than twice its breadth.
Therefore, length of rectangular plot is $(2x+1)$ metres
Area of rectangle $=$ length $\times$ breadth
$\Rightarrow 528=x(2x+1)$
$\Rightarrow 528=2x^2+x$

$\Rightarrow 2x^2+x^{-528}=0$

This is required Quadratic Equation.
(ii) Let two consecutive numbers be x and $(\mathrm{x}+1)$.

It is given that $x(x+1)=306$
$\Rightarrow x^2+x=306$
$\Rightarrow x^2+x^{-306}=0$

This is the required Quadratic Equation.
(iii) Let present age of Rohan $=$ x years

Let present age of Rohan's mother $=(x+26)$ years
Age of Rohan after 3 years $=(x+3)$ years
Age of Rohan's mother after 3 years $=x+26+3=(x+29)$ years

According to given condition:
$(x+3)(x+29)=360$
$\Rightarrow x^2+29x+3x+87=360$
$\Rightarrow x^2+32x-273=0$

This is the required Quadratic Equation.

(iv) Let speed of train be $x \mathrm{~km}/\mathrm{h}$

Time taken by train to cover $480 \mathrm{~km}=\frac{480}{x}$ hours
If, speed had been $8 \mathrm{~km}/\mathrm{h}$ less then time taken would be $\frac{480}{x-8}$ hours

According to given condition, if speed had been $8 \mathrm{~km}/\mathrm{h}$ less then time taken is 3 hours less.
Therefore, $\frac{480}{x-8}-\frac{480}{x}=3$

$\Rightarrow 480(x-x+8)=3x(x-8)$
$\Rightarrow 3840=3x^2-24x$
$\Rightarrow 3x^2-24x-3840=0$

Dividing equation by 3 , we get
$\Rightarrow x^2-8x-1280=0$

This is the required Quadratic Equation.

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Ex 4.2 Question 1.

Find the roots of the following Quadratic Equations by factorization.

(i) $x^2 - \mathbf{3}x - \mathbf{10} = \mathbf{0}$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Answer.

(i) $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x-5)(x+2) = 0$

$\Rightarrow x = 5, -2$

(ii) $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0$

$\Rightarrow (2x-3)(x+2) = 0$

$\Rightarrow x = \frac{3}{2}, -2$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x^2+2x+5x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x(x+\sqrt{2})+5(x+\sqrt{2})=0$

$\Rightarrow (\sqrt{2}x+5)(x+\sqrt{2})=0$

$\Rightarrow x=\frac{-5}{\sqrt{2}},-\sqrt{2}$

$\Rightarrow x=\frac{-5}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}},-\sqrt{2}$

$\Rightarrow x=\frac{-5\sqrt{2}}{2},-\sqrt{2}$

(iv) $2x^2-x+\frac{1}{8}=0$

$\Rightarrow \frac{16x^2-8x+1}{8}=0$

$\Rightarrow 16x^2-8x+1=0$

$\Rightarrow 16x^2-4x-4x+1=0$

$\Rightarrow 4x(4x-1)-1(4x-1)=0$

$\Rightarrow (4x-1)(4x-1)=0$

$\Rightarrow x=\frac{1}{4},\frac{1}{4}$

(v) $100x^2-20x+1=0$

$\Rightarrow 100x^2-10x-10x+1=0$

$\Rightarrow 10x(10x-1)-1(10x-1)=0$

$\Rightarrow (10x-1)(10x-1)=0$

$\Rightarrow x=\frac{1}{10},\frac{1}{10}$

Ex 4.2 Question 2.

Solve the following problems given:

(i) $x^2-45x+324=0$

(ii) $x^2-55x+750=0$

Answer.

(i) $x^2-45x+324=0$

$\Rightarrow x^2-36x-9x+324=0$

$\Rightarrow x(x-36)-9(x-36)=0$

$\Rightarrow (x-9)(x-36)=0$

$\Rightarrow x=9,36$

(ii) $x^2-55x+750=0$

$\Rightarrow x^2-25x-30x+750=0$

$\Rightarrow x(x-25)-30(x-25)=0$

$\Rightarrow (x-30)(x-25)=0$

$\Rightarrow x=30,25$

Ex 4.2 Question 3.

Find two numbers whose sum is 27 and product is 182 .

Answer.

Let first number be x and let second number be $(27-x)$

According to given condition, the product of two numbers is 182 .

Therefore,

$x(27-x)=182$

$\Rightarrow 27x-x^2=182$

$\Rightarrow x^2-27x+182=0$

$\Rightarrow x^2-14x-13x+182=0$

$\Rightarrow x(x-14)-13(x-14)=0$

$\Rightarrow (x-14)(x-13)=0$

$\Rightarrow x=14,13$

Therefore, the first number is equal to 14 or 13

And, second number is $=27-x=27-14=13$ or Second number $=27-13=14$

Therefore, two numbers are 13 and 14.

Ex 4.2 Question 4.

Find two consecutive positive integers, sum of whose squares is 365 .

Answer.

Let first number be x and let second number be $(x+1)$
According to given condition,
$x^2+(x+1)^2=365$
$\{(a+b)^2=a^2+b^2+2ab\}$
$\Rightarrow x^2+x^2+1+2x=365$
$\Rightarrow 2x^2+2x-364=0$

Dividing equation by 2
$\Rightarrow x^2+x-182=0$

$\Rightarrow x^2+14x-13x-182=0$
$\Rightarrow x(x+14)-13(x+14)=0$

$\Rightarrow (x+14)(x-13)=0$
$\Rightarrow x=13,-14$

Therefore, first number $=13$ \{We discard -14 because it is negative number)
Second number $=x+1=13+1=14$

Therefore, two consecutive positive integers are 13 and 14 whose sum of squares is equal to 365.

Ex 4.2 Question 5.

The altitude of right triangle is 7 cm less than its base. If, hypotenuse is 13 cm. Find the other two sides.

Answer.

Let base of triangle be $x\text{ cm}$ and let altitude of triangle be $(x-7)\text{cm}$
It is given that hypotenuse of triangle is 13 cm

According to Pythagoras Theorem,
$(13)^2=x^2+(x-7)^2$
$\Rightarrow 169=x^2+x^2+49-14x$ $\quad$ $\left[\text{Since, }(a+b)^2=a^2+b^2+2ab\right]$
$\Rightarrow 169=2x^2-14x+49$
$\Rightarrow 2x^2-14x-120=0$

Dividing equation by 2
$\Rightarrow x^2-7x-60=0$
$\Rightarrow x^2-12x+5x-60=0$
$\Rightarrow x(x-12)+5(x-12)=0$
$\Rightarrow (x-12)(x+5)=0$
$\Rightarrow x=-5,12$

We discard $x=-5$ because length of side of triangle cannot be negative.
Therefore, base of triangle $=12\text{ cm}$
Altitude of triangle $=(x-7)=12-7=5\text{ cm}$

Ex 4.2 Question 6.

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

Answer.

Let cost of production of each article be Rs x
We are given total cost of production on that particular day $=$ Rs 90
Therefore, total number of articles produced that day $=90/x$
According to the given conditions,
$x=2\left(\frac{90}{x}\right)+3$
$\Rightarrow x=\frac{180}{x}+3$
$\Rightarrow x=\frac{180+3x}{x}$
$\Rightarrow x^2=180+3x$
$\Rightarrow x^2-3x-180=0$
$\Rightarrow x^2-15x+12x-180=0$
$\Rightarrow x(x-15)+12(x-15)=0$
$\Rightarrow (x-15)(x+12)=0\Rightarrow x=15,-12$

Cost cannot be in negative, therefore, we discard $x=-12$
Therefore, $x=$ Rs 15 which is the cost of production of each article.
$\text{Number of articles produced on that particular day }=\frac{90}{15}=6$

NCERT Solutions for Class 10 Maths Chapter 4: Quadratic Equations

Ex 4.3 Question 1.

Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.
(i) $2x^2 - 3x + 5 = 0$
(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
(iii) $2x^2 - 6x + 3 = 0$

Answer.

(i) $2x^2 - 3x + 5 = 0$
Comparing this equation with general equation $cx^2 + bx + c = 0$,
We get $a = 2, b = -3$ and $c = 5$
Discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5)$
$= 9 - 40 = -31$

Discriminant is less than 0 which means equation has no real roots.
(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with general equation $cx^2 + bx + c = 0$,
We get $a = 3, b = -4\sqrt{3}$ and c $= 4$
Discriminant $= b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$
$= 48 - 48 = 0$

Discriminant is equal to zero which means equations has equal real roots.

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots,
$x = \frac{4\sqrt{3} \pm \sqrt{0}}{6} = \frac{2\sqrt{3}}{3}$

Because, equation has two equal roots, it means $x = \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$
(iii) $2x^2 - 6x + 3 = 0$

Comparing equation with general equation $cx^2 + bx + c = 0$,
We get $a = 2, b = -6$, and $c = 3$
Discriminant $= b^2 - 4ac = (-6)^2 - 4$
$= 36 - 24 = 12$

Value of discriminant is greater than zero.
Therefore, equation has distinct and real roots.
Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots,

$$x = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$
$$\Rightarrow x = \frac{3 \pm \sqrt{3}}{2}$$
$$\Rightarrow x = \frac{3 + \sqrt{3}}{2}, \frac{3 - \sqrt{3}}{2}$$

Ex 4.3 Question 2.

Find the value of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2+kx+3=0$
(ii) $kx(x-2)+6=0$

Answer.

(i) $2x^2+kx+3=0$
We know that quadratic equation has two equal roots only when the value of discriminant is equal to zero.

Comparing equation $2x^2+kx+3=0$ with general quadratic equation $cx^2+bx+c=0$, we get $a=2, b=k$ and $c=3$

Discriminant $=b^2-4ac=k^2-4^{(2)}(3)=k^2-24$
Putting discriminant equal to zero
$k^2-24=0 \Rightarrow k^2=24$
$\Rightarrow k=\pm\sqrt{24}=\pm 2\sqrt{6}$
$\Rightarrow k=2\sqrt{6},-2\sqrt{6}$
(ii) $kx(x-2)+6=0$
$\Rightarrow kx^2-2kx+6=0$

Comparing quadratic equation $kx^2-2kx+6=0$ with general form $cx^2+bx+c=0$, we get $a=k, b=-2k$ and $c=6$
$\text{Discriminant } =b^2-4ac=(-2k)^2-4(k)(6)$
$=4k^2-24k$

We know that two roots of quadratic equation are equal only if discriminant is equal to zero.
Putting discriminant equal to zero

4 k^2-24 k=0

$\Rightarrow 4k(k-6)=0 \Rightarrow k=0,6$

The basic definition of quadratic equation says that quadratic equation is the equation of the form $cx^2+bx+c=0$, where $a\neq 0$.

Therefore, in equation $kx^2-2kx+6=0$, we cannot have $\mathrm{k}=0$.
Therefore, we discard $\mathrm{k}=0$.
Hence the answer is $\mathrm{k}=6$.

Ex 4.3 Question 3.

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800\ \mathrm{m}^2$? If so, find its length and breadth.

Answer.

Let breadth of rectangular mango grove $=x$ metres
Let length of rectangular mango grove $=2x$ metres
Area of rectangle $=$ length $\times$ breadth $=x\times 2x=2x^2m^2$
According to given condition:
$\Rightarrow 2x^2=800$
$\Rightarrow 2x^2-800=0$
$\Rightarrow x^2-400=0$

Comparing equation $x^2-400=0$ with general form of quadratic equation $cx^2+bx+c=0$, we get $a=1, b=0$ and $c=-400$
$\text{Discriminant } =b^2-4ac=(0)^2-4(1)(-400)=1600$

Discriminant is greater than 0 means that equation has two disctinct real roots.
Therefore, it is possible to design a rectangular grove.

Applying quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,
$x=\dfrac{0\pm\sqrt{1600}}{2\times 1}=\dfrac{\pm 40}{2}=\pm 20$
$\Rightarrow x=20,-20$

We discard negative value of x because breadth of rectangle cannot be in negative.
Therefore, $x=$ breadth of rectangle $=20$ metres
Length of rectangle $=2x=2\times 20=40$ metres

Ex 4.3 Question 4.

Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer.

Let age of first friend $=x$ years
then age of second friend $=(20-x)$ years
Four years ago, age of first friend $=(x-4)$ years

Four years ago, age of second friend $=(20-x)-4=(16-x)$ years
According to given condition,

$\Rightarrow(x-4)(16-x)=48$

$\Rightarrow 16x-x^2-64+4x=48$

$\Rightarrow 20x-x^2-112=0$

$\Rightarrow x^2-20x+112=0$

Comparing equation, $x^2-20x+112=0$ with general quadratic equation $cx^2+bx+c=0$, we get $a=1, b=-20$ and $c=112$

$\text{Discriminant } =b^2-4ac=(-20)^2-4(1)(112)=400-448=-48<0$

Discriminant is less than zero which means we have no real roots for this equation.
Therefore, the give situation is not possible.

Ex 4.3 Question 5.

Is it possible to design a rectangular park of perimeter 80 metres and area $400 \mathrm{~m}^2$. If so, find its length and breadth.

Answer.

Let length of park $=x$ metres
We are given area of rectangular park $=400m^2$
Therefore, breadth of park $=\frac{400}{x}$ metres $\{$ Area of rectangle $=$ length $\times$ breadth $\}$
Perimeter of rectangular park $=2($ length + breath $)=2\left(x+\frac{400}{x}\right)$ metres

We are given perimeter of rectangle $=80$ metres
According to condition:

$\Rightarrow 2\left(x+\frac{400}{x}\right)=80$

$\Rightarrow 2\left(\frac{x^2+400}{x}\right)=80$

$\Rightarrow 2x^2+800=80x$

$\Rightarrow 2x^2-80x+800=0$

$\Rightarrow x^2-40x+400=0$

Comparing equation, $x^2-40x+400=0$ with general quadratic equation $ax^2+bx+c=0$, we get $a=1, b=-40$ and $c=400$

$\text{Discriminant } =b^2-4ac=(-40)^2-4(1)(400)=1600-1600=0$

Discriminant is equal to 0 .
Therefore, two roots of equation are real and equal which means that it is possible to design a rectangular park of perimeter 80 metres and area $400 \mathrm{~m}^2$.

Using quadratic formula $x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ to solve equation,
$x=\frac{40 \pm \sqrt{0}}{2}=\frac{40}{2}=20$

Here, both the roots are equal to 20 .
Therefore, length of rectangular park $=20$ metres
$\text{Breadth of rectangular park } =\frac{400}{x}=\frac{400}{20}=20 \mathrm{~m}$